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# A SUGGESTED PLANNING PROCEDURE WITH SCARCITY IN A CENTRALLY PLANNED ECONOMY\*

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This paper examines, after a brief review of planning procedures, a scarcity restricted - central planning model. An initial feasible programme is assumed to exist. It was shown that the model (Novozhilov, 1970) can be extended into a dynamic planning model whereby an improving macro plan is achieved with every iteration. Procedure described, has the primary desirable properties of planning routine: it is well-defined, convergent and furthermore labour - value based. It is closely associated with the multi - echelon type of planning procedures.

#### 1. Introduction

The central planning of production is no longer, apparently, within the theoretical or practical domain of socialist economies alone. In varying degrees the industrialized capitalist nations and the poor of the world have come to realize that a conscious intervention with the "automatic" workings of "invisible hand" is overdure. This realization has everywhere brought about important changes in economic insitutions and policy. Increasingly reliance is placed upon central planning. Experience and necessity have given birth to the theory of planning and of economic organization. In the Western World, as well as in the Eastern, a growing body of literature on planning methods and procedures has emerged. This, needless to say, is quite indepen-

(\*) Presented to the Third Annual Operations Research Congress, İzmir, 1977. (\*\*) Assistant Professor, Middle East Technical University. dent of the recent swing to the traditional economic policies observed in most of the advanced capitalist countries. Planning Organizations are indispensable, nevertheless.

## 2. A Brief History of Planning Procedures :

Most of the early material concerning economic planning was descriptive or dealt with institutional requirements. Concrete theoretical problems have gradually arisen, and born out of socialist planning experience in combination with a recognition of the importance of different technological alternatives to production.

The first systematic essay on economic planning was undertaken by Barone (1908). He sought to show that a central plan should obey the same marginal conditions as would be generated within a purely competitive market economy. Further important contributions on the subject have been made in the last thirty-five years. O. Lange's work (1938) certainly constitutes a cornerstone in the formulation of planning procedures.

Lange began what it is still called "the economic theory of socialism" in the literature. His work may be classified as a theory of "market socialism" a type of model which was taken up later by Arrow and Hurwicz (1960) and Malinvaud. (1967).

Lange was trying to counter Professor von Mises' argument that a rational allocation of resources is impossible within a socialist state because the public ownership of the means of production would wipe out the market for capital goods. Lange's response to this contention was that von Mises had confused the nature of prices and of markets. He contended that if prices were looked upon only as "terms on which alternatives are offered," markets of a certain type could exist under socialism and adequate planning procedures could be defined. This view of prices is the forerunner of the prices of today's programming models. Prices are thus viewed now as accounting prices or provisional valuations for the purpose of allocating resources.

The existence of the market in the sphere of production, according to Lange, depends upon an independence of the socialist accounting prices from actual market prices. That is, the

"parametric" function of the acounting prices must be observed by the managers of production units. Once the prices are fixed (in the accounting sense) by the Central Planning Board (CPB), the discrepancy from the actual market price is immediately established by an objective equilibrium condition specified by the CPB. Any price different from the equilibrium price must yield a surplus or shortage of the commodity at the end of the accounting period. From this consideration, Lange seeks to show that accounting prices in a socialist economy, far from being arbitrary, may have quite the same decisive character as the market prices within the competitive market. Any mistake made by the CPB would reveal itself in the form of a surplus or shortage of the particular commodity, and hence, prices could be corrected so as to maintain a smooth running, equilibrium production. This is the "trial and error" method of Lange which was based on the Walrasian "tatonnement". In fact, the trial and error method can be found in earlier studies. Barone (1908) and Taylor (1928) are the ones who have discussed the method.

Lange - Lerner type model, along with technically sophisticated Arrow-Hurwicz and Malinvaud models, belongs to the family of tatõnnement procedures. We are not going to look further into these models. However, as far as planning theory is concerned, one important drawback of tatõnnement procedures must be mentioned. This drawback is their ommision of the vital and dynamic part of the economy, the technical structure. The only requirement of tatõnnement procedures is the calculation of net demands without recourse to the technology. For this reason, tatõnnement procedures cannot seriously be viewed as planning models.

Another important family of planning procedures builds heavily upon the decomposition of macro-programmes and hence hierarchical mode of economic organization. Characteristically, a large programming problem is decomposed into smaller and managable pieces and then they are tied together with a master programme. The best example of this is Dantzig-Wolfe (1960) and Dantzig (1963) model. Their problem involves independent production units with their particular technical restrictions and capabilities. These independent units are brought together in the objective function with their trade relations.

Malinvaud's liner model involving "Leontief - Samuelson" technology bears a close resemblance to the Dantzig - Wolfe Decomposition principle. In both models, final programmes (output vectors) can be represented as linear combination of the other vectors. The problem turns out to be to pick appropriate weights to determine the optimal mix of net output.

Inspired by the Malinvaud's work, there is another important work by Weitzman (1970). His procedure, as author claims, can be viewed as the dual of Dantzig-Wolfe and Malinvaud approaches in many respects. While D-W-M approach starts from prices and production plans improve with every step, Weitzman's procedure reaches the optimal plan from the other end. That is, the CPB becomes progressively less optimistic about the attainable level of production in the process of planning.

A complementary piece of work to that of Dantzig - Wolfe was introduced by Almon (1963). His work presupposes that technological matrices are hard to compose and not explicitly known to the center. But the CPB has the record of feasible programs of previous years if not the complete spectrum of potentialities. The subprograms involved in the selection of appropriate combinations of previous programs are known to the CPB. The procedure will start with the CPB calculating the prices (simplex multipliers). Each unit will be told to minimize costs at the new set of prices. New techniques with the lowest relative cost, proposed by the units, will be included within the basis and a new set of prices will be calculated. Almon's work improve on Dantzig-Wolfe model by suggesting additional indices be met by units which would help cut down the number of iterations.

There are several recent studies on planning procedures and the theory of resource allocation. The majority of these models start from an Arrow-Hurwicz type of construct and try to generalize the model to the well-known case of increasing returns. (Aoki, 1971). Yet slightly different type of application of the resource allocation theory can be seen in a recent work of Radner and Groves (1972). They develop a theory of economic organization and team decisions from the informational efficiency of "price" and "demand" messages in a resource allocation mechanism.

Principal work on efficiency of information was undertaken by Hurwicz (1971). He compared the resource allocation mechanisms with respect to their informational requirements. If a given process requires lesser information in some respect, then it is viewed as "informationally efficient". Mount and Reiter (1974) has investigated the general framework from which the embodied information and the informational requirements of a process could be determined. This line of analysis is beyond our scope in this paper, but this turn in the theory of economic planning towards the traditional topics of the theory of resource allocation is worth noticing. Very promiment on this line is the work of Heal (1973) and an excellent book written in Turkish by Ersel (1978).

# 3. East European Studies on Planning Procedures :<sup>(1)</sup>

The basic instrument of socialist planning is the material balances. Despite several reforms, proposals and technically sophisticated models, the method of material balances is at the core of socialist planning.<sup>(2)</sup> The structure and the administrative mechanism of planning differ within the various East European countries; yet, the task of elaborating a set of consistent balances involve the same basic problems in every centrally planned economy. Hence, the planning methods and procedures reflect the problems of balancing the physical allocation plan.

The planning process with material balances is a multiechelon operation. The informational exchange, commands and messages do take place between three separete entities: 1. The CPB, 2. Ministries, 3. Enterprises. Before the beginning of the plan-period, the CPB, taking into account the latest production statistics, estimates of the productive capacity and labor force, prepares a preliminary balance of essential materials. Then, basing calculations upon these estimates, it hands down tentative targets to the ministries. Each ministry, in turn, assigns specific

<sup>(1)</sup> Material reviewed in this section refers to secondary sources unless the work is done in English or translated subsequently. Hence, proper references to most of the authors could not be made. Zauberman (1967) and Ellman (1973) are the secondary sources of reference which we have extensively used.

<sup>(2)</sup> For an exposition of the method of material balances, Montias (1959) is still the best source.

and dissagregated targets to the enterprises under their supervision. Each enterprise is expected to calculate the material inputs in order to meet the specified targets. From this point on, a reverse operation starts. The formal requests of enterprises are aggregated and checked with the latest output figures and with established norms regulating the maximum permissible expenditure of materials per unit of output. The ministry turns over its plans to the CPB for further aggregation at the macro level. At the second stage, more precise and detailed plans are prepared at ministerial level, modifying and complementing the control figures they have received earlier. These simultaneous adjustments of supply and demand continue until total demand and supply matches. Before all balances are simultaneously closed, however, it is often necessary to go through the same procedures several times. The material balances technique then belongs to the family of iterative methods.

As practised in socialist countries, the iterative solution to final balances is a deterministic approach. The optimality characteristics of the material balances, however, depends upon the selection of the most efficient activities. If planners assume that the starting basis is an optimal basis, the iterative solution not only yields correct and consistent estimates but optimal proportions.

Parallel to the basic material balances, the traditional iterative model operates on the basis of the aggregation and disaggregation of variables and constants of an input-output system. A good representative of these family of models is Yu. N. Gavrilyets' construct. In this model optimization is carried out with respect to a criterion of "optimal error" (optimal in a pre-defined sense) resulting from the aggregation-disaggregation process. The error term is added to the objective function of the overall program. The solution to the micro-problem is controlled by the macroproblem. In aggregation by products, the micro - problems are solved and a convergence of efficiency prices for resources is sought. In aggregation by resources, however, the convergence of macro outputs is important. Zauberman (1967) notes that experience indicates the comparability of the convergence properties of the model and the simplex method.

Another important class of model is distinctive because of its close resemblance to Dantzig-Wolfe approach. The most distinctive and original contribution is the model of two-level planning by two Hungarian economists, J. Kornai and Th. Liptak (1965). They formulate the problem as one of linear programming in which a large macro-problem may be decomposed into subproblems that can be solved by mutually independent sectors coordinated by the center. Kornai-Liptak calls the master program "overall central information problem". This framework is transformed into a two-level problem in which transactions take place between sectors and the CPB in a coordinated manner. In a sense, sub-problems are derived (as is the case with Dantzig-Wolfe) from the overall central information problem. The difference between the Kornai-Liptak model and the others is the game-theoretic approach to the solution of two-level problem. The players are CPB on the one side and the team of the sectors on the other. The strategy of the CPB is the set of feasible allocation patterns and the strategy of the sectors is the feasible shadow prices.

In place of a direct solution, authors substitute a fictitious game which simulates, to some degree, a common planning procedure. Initially, the CPB assigns resources to the sectors. On the basis of this allocation, sectors evaluate the scheme by means of shadow prices and report back to the CPB. The center, then, issues a new programme revised in the light of the fresh information from the units. This completes the cycle. Iteration is carried out to the desired degree of accuracy which is the difference between the feasible and optimum programmes.

A distinctive feature of the Kornai-Liptak construct was pointed out by Zauberman. No prices are determined from the dual by CPB. They are set at sectoral level while the CPB indirectly exercises control on readjustments of the tasks. This means that prices, although not imputed by the CPB, are implicit in central directives.

On Dantzig-Wolfe lines, there are two important models by Soviet economists. Volkonskiy of the Cybernetics Institute of Ukranian Academy of Sciences has elaborated the traditional multi-echelon material balances method (Zauberman, 1967) into

an optimal planning procedure. In this model each unit faces the constraints with regard to its limited resources and its objective function given from the superior echelon. Shadow prices are boundaries defining the allowable limits of violation of some constraints from which the objective function is derived. The objective function, then, becomes of a paramount importance for the CPB, because it is the most obvious control instrument of the performance of units. Planning routine starts with the CPB solving the master program constructed from adjusted past data. Shadow prices and the nature of scarcities are sent down to the units and the lower echelons then offer plans of inputs and outputs satisfing their objective function. New proposals will promulgate a new set of prices and a new round will start rolling again. The iteration will be terminated when supply and demand equilibrium prices are satisfactorily approximated.

The characteristic of Volkonskiy construct is the gain in the speed of adjustments as compared to the Dantzig-Wolfe due to the fact that constraints which are not effective are left out in the next iteration.

A second model is by Pugachev. It has beer undertaken under the auspices of TSEMI (The Central Economic Mathematical Institute of the USSR Academy of Sciences) in 1966. This study was put forward as proposal to overcome the theoretical and organizational difficulties faced during the planning experience in erlier years relative to the theory of the optimally functioning economy. In 1969, TSEMI put forward another proposal on much the same lines. Like Volkonskiy's model, this is a multiechelon planning procedure (enterprises, industries and the CPB) with both prices and quantities to be adjusted. This method cuts down the number of iterations considerably by aggregating the constraints. One linear constraint is the capabilities of each unit. A macro optimum is determined with respect to the quadratic objective function and the quantities of branch outputs and input prices.

The interesting feature of the two Soviet models (Volkonskiy and Pugachev) is that they try to formalize planning with "sliding horizon". They reject the dichotomy between of short and long-run planning. Instead, they dynamize the system by

iterating with respect to the latest information available. The shadow prices calculated from the dual indicate the direction of technical progress and also provide the next informational basis for planning on the assumption that units are ordering new means of production based upon these prices. Naturally, the technical structure and data will be altered for the next iteration of the system.

We must mention an outstanding model by a Polish economist, K. Porwit (1967). He effectively combined the material balances method with modern programming. Prowit's model<sup>(3)</sup> starts when the CPB derives the sectoral output targets from the material balances. In some principal products these targets are physical in accordance with the basic balances. Besides physical targets, there are interindustry delivery targets weighted with constant prices. The model derives its input limitations from the material balances of main products. The CPB also supplies the shadow prices for specific categories of capital goods in short supply. Each sector is expected to minimize its use of labour and other resources supplied from outside. Within this model, each sector calculates its own dual prices on the basis of fixed prices supplied by the CPB. This will provide the best allocation of external deliveries to the sectors. The dual conceives of the value of the national product as maximand. One can see that Porwit's model is quite comparable to that of Novozhilov —which we shall see in the next section— in many respects.

After this brief review of the planning procedures, one could classify them under two categories : 1. Tatõnnement procedures, 2. Multi-echelon, decomposed models. We hold that the tatõnnement procedures do not provide an appropriate framework for efficient macro-plans, because they do not incorporate the vital influence of the technical structure of the economy. All serious planning work necessitates a rather detailed picture of the economy. The productive capabilities of the sectors, the availability of the means of production, etc., must be accounted for in detail. Hence, Lange-Lerner type models and their derivatives (Arrow-Hurwicz) cannot be regarded as planning models in the strict sense of the word.

<sup>(3)</sup> Four full mathematical exposition of Portwit's model see chapters I and II including the appendices.

The Dantzig-Wolfe type models, on the other hand, provide a satisfactory framework for planning procedures. Most of the important work in the Western World and, East Europe have advanced on Dantzig-Wolfe lines. The idea of "decomposition" seems to capture the multiechelon nature of the socialist planning experience.

## 4. The Scope of the Proposed Models :

It is clear that the optimal plan is a conditional phenomenon. It draws its optimality characteristics from the prevailing scarcities and from the technological structure of the economy. At present the technological superstructure and the state of existing scarcities may be viewed as arising historically from an arbitrary rule of allocation or a lopsided price structure. In either case the results is the misuse of the resources and the waste of the means of production.

It is in connection with misallocation that the most forbidding theoretical obstacles to corrective planning are faced. The corrective plan must first of all be constructed of materials drawn from an initial economic setting which is anything but optimal. Because of such a complex situation, common to planned economies, "first approximation" price programming results are bound to be distorted by prevailing cost-price relations and by the nature of scarcities. Given distortions of the basic data, what change is there that they will provide the basis for an optimal price structure? To what extent can programming prices be thought of as elements of the optimal plan?

These questions raise issues of logical circularity and simultaneity. We know that an optimal price structure can only be obtained from an optimal physical allocation programme. But the latter, in turn, requires an optimal physical allocation programme. But the latter, in turn, requires an optimal price structure as its own precondition. The only way out of this circularity is an iterative procedure that enables us to trace the path of optimality adjustments of successively optimal plans. Furthermore, the possibility of such iterative procedures provides a justification for planners starting from any arbitrary set of prices or of factor proportions because, whatever the initial resources

and the state of technology, an optimal corrective plan may be more closely approximated with every iteration of the system. This section, then, attempts to justify a planning procedure built upon Novozhilov's optimal planning model<sup>(4)</sup> (Novozhilov, 1970).

Specifically, we shall try to expand on the Novozhilov model by converting it into a dynamic planning procedure while utilizing its characteristic features. As it stands, Novozhilov's model is a static, conditionally optimizing model. Its static character comes from its fixed technology set. The conditional optima is viewed as a product of the technology set and from the priori valuations which are embedded in its unit costs. The solution to both of these difficulties of static planning is a planning procedure into which the new activities can be introduced and, consequently, the price structure progressively altered.

We should point out, however, that no practical policy recommendations (and organizational suggestions) are intended in this paper. The overriding concern is not the actual planning procedures and mechanism itself but only the logical possibility for such procedures extending from the planning model we will be considering. Of course it is quite possible for there to be a number of distinct procedures which can achieve the same objectives. Some may be highly efficient in utilizing information and others may have various speeds of adjustment built into them. For our purpose, in this paper, however the existence of any such procedure suffices to make our point no matter how inefficient and clumsy it may be.

Needless to say, the planning routines we will be describing do not correspond with actual planning practices in a mixed economy. Neither do they correspond with, much less explain the techniques used within specific socialist countries. Primarily, it is of some pedagogical value for it explains in a simple manner a decision-making process of a socialist economy where the means of production are owned by the state. In principle, such

<sup>(4)</sup> Novozhilov's model was originally formulated in 1967 but its english translation has reached the Western reader in 1970. Elements of his model, however, can be traced back to his book length essay written in late filfties and edited in Nemchinov (1964).

planning routines may shed some light on the bourgeouis planning experience and development plans of some poor nations.

This paper describes a mechanistic view of planning and planning routines from the point of technical choices and allocation. This, of course, is not to deny the importance of other socio-economic aspects of planning. In practice, the preparation of plans strongly suggests the study of collective preferences, the income distribution, workers' participation in decision-making and all phases of plan operations as well as financing of production units. Such important topics of planning will not be discussed here but we will rather limit ourselves to the possible incorporation of technical choices into overall improvement of **successive** plans.

The planning procedure by which improving plans can be generated is a dynamic process, and as such may allow for a changing structure of technology and cost-price relations. In actuality planning does not take place instantaneously, but only slowly and painstakingly through time. Nevertheless, for our purposes, the actual time elapsing from the beginning of preparation of the plan to the end of the procedure is unimportant. In other words, the time elapsed during the process is fictitious. Once the theory assures us that such a procedure is feasible, the actual shortening of time elapsed between formulation and realization of objectives involves the planners in dealing with the organizational features of the economy with which we need not concern ourselves here.

Before going into the description of planning procedure, it will be useful first to restate the basic elements of the Novozhilov model.

#### 5. Novozhilov Model - A Restatement of Important Results :

The Novozhilov model deals with a macro-plan in which total effort to meet the predetermined goals is to be minimized : The minimum amount of labour time (socially necessary labour) is to be expended to meet a final product goal. Hence, primal and dual programmes are formulated as follows ;

## PRIMAL:

$$\sum_{I} q_{i}^{I} \geq q_{i} \quad (i = 1, 2, ..., n)$$

DUAL :

 $\begin{array}{ll} \text{Maximize } \sum\limits_{J} r_{j} (-Q_{j}) + \sum\limits_{j} p_{i} q_{i} (i = 1, 2..., n; j = 1, 2, ..., m) \\ \text{Subject to:} \quad p_{i} \leqslant c_{i}^{I} + \sum\limits_{J} a_{i}^{I} jr_{j} (i = 1, 2..., n) \end{array}$ 

where :

- $c_{i}^{1}$  = Full labour costs for producing i th product with process l per period of plan.
- $a^{i}_{ij}$  = Expenditure of resource j per unit of output i with l th process.

 $q_{i}^{1}$  = The amount of i th final product produced with process 1.

 $Q_j$  = The amount of resource of type j available at the beginning of the planned period.

 $q_i$  = Demand for final product i.

 $r_{i}$  = The norm of effectiveness of utilization of the j th resource.

 $P_i$  = Consumption valuation of the i th product.

From the model following results are immediate.

1. 
$$r_j=0$$
 when  $\sum_{i,I} a_{ij}^I q_i^I < Q_j$ ,  $r_j=0$  when  $\sum_{i,I} a_{ij}^I q_i^I = Q_j$ 

2. 
$$q_i^1 = 0$$
 when  $p_i^I < c_i^I + \Sigma a_{ij}^I r_j$ ,  $q_i^I > 0$  when  $P_i^I = c_i^I + \Sigma a_{ij}^I r_j$ 

3. 
$$p_i=0$$
 when  $\sum_{i} q_i^1 > q_i$ ,  $p_i > 0$  when  $\sum_{i} q_i^I = q^i$ 

4. 
$$\sum_{i,i} c_i^I q_i^I = \sum_i p_i q_i - \sum_j r_j Q_j$$

(From duality theorem)

- 5. From 1 and 2 it can be confirmed once again that when scarcities are assumed away,  $r_j = 0$  and  $p_i = c_i$ ,  $q_i^1 > 0$ , optimal price is equal to total labour costs.
- 6. When plan is optimal,  $\sum_{j} r_{j} Q_{j} = \sum_{i,j,l} a_{ij}^{I} r_{j} q_{j}^{I}$ .

Because first constraint of the primal programme will hold as an equality. We rule ovt the case where some  $r_{\rm j}=0$ .

## a) Assumptions of the Suggested Planning Procedure :

We have stated that our model will be built upon Novozhilov's construct as just described. It will include its basic assumptions, namely, linear technologies, fixed final demands and the existence of scarcities. In addition to those, I shall include three more :

Assumption 1: There are two different planning bodies 1 — The Central Planning Board (CPB) 2 — Production units. Production units are identified with specific activities and there are as many production units as activities.

Assumption 2: There exists a feasible plan at the initial plan period "t" (t = 0,1,2,...T) That is, the production set,  $Q^s$ , is capable of producing the predetermined vector of goods, q $\in$ Q,

#### $Q^{s} = (q : Aq \leq Q, q > 0)$

Assumption 3: The target output (predetermined vector of goods) will remain fixed throughout the planning cycle.  $q_{i}^{t} = q_{i}^{t+1} = q_{i}^{t+T}$ .

b) Definitions :

**Definition 1:** A plan is called "optimal" (Novozhilov model) if there are no other plans that would produce the predetermined basket of goods with less labour time expended.

**Definition 2**: A plan is feasible if the restriction of primal problem are obeyed.

**Definition 3:** A planning procedure may be called "welldefined" if the plan is a feasible programme and proce and quantity signals are such that at every step the programme is determined.

**Definition 4**: A plan period is a time span in which all steps are completed.

Definition 5: A plan is said to be improving if,

$$\sum_{i,I} c_i^{I,t+I} \quad q_i^{It+I} \leq \sum_{I;I} c_i^{It} \quad q_i^{It} \quad (t=0,1,2,...,T)$$

**Definition 6:** A plan is called "convergent" if it tends to a lower value for socially necessary expenditures when iterations are carried indefinitely. Let us call the value of the program S and the lowest bound for the value  $S^*$ , then,  $\lim S = S^*$ .

 $t \rightarrow \infty$ 

# 6. The Importance of Scarcity Valuations as a tool of Minimization :

Let us call the Lagrangian function generated from the primal problem, L:

$$L = \sum_{i,I} c_i^I q_i^I - \hat{r} \quad (\sum_{i,I} a_{ij}^I q_i^I - Q_j) - \hat{p} \quad (\sum_{I} q_i - q_j)$$

where rand p are Lagrange multipliers.

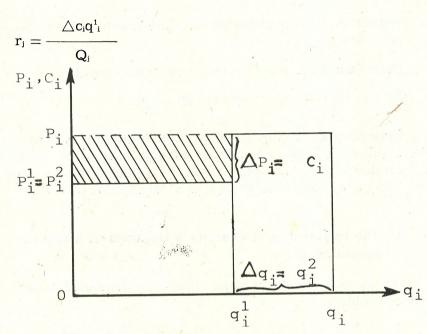
From the Lagrangian an immediate identifaciton of scarcity valuations emerges.  $r_j$ , associated with j th scarce resource, is equal to the partial derivative of the Lagrangian with respect to the quantity of j th scarce resource.

$$rac{\partial L}{\partial Q_j} = r_j \ (j = 1, 2, ..., m)$$

It can be interpreted as the contribution of an extra unit of j th resource to the overall program.

This identification can be illustrated on a diagram. As before, we shall consider only two activities involving in the production of i th product. Activity 1 will produce  $q^{1}_{i}$  portion of the output and Activity 2 will produce the rest. The crossed rectangle in the diagram represents the quantity  $r_{j}Q_{j}$ . Denoting  $p_{i}-p^{1}_{i}$  by  $\Delta p_{i}$ , this area will also be equal to  $\Delta p_{i}q^{1}_{i} = \Delta c_{i}q^{1}_{i}$ . From the equality  $r_{j}Q_{j} = \Delta c_{i}q^{1}_{i}$ , we have

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The scarcity valuation of the j th resource is clearly equal to the per unit share of the j th resource in the differential rent or feedback costs it occasioned. Since  $Q_j$  is fixed,  $r_j$  will be positively related to the unit production cost and quantity produced with the restricted activity (Act.1). This important feature of  $r_j$  will enable us under certain circumstances to initiate a planning algorithm. We shall consider two distinct cases : 1 — They production units are assigned specific quotas. 2 — They have no quotas.

## a) Planning Procedure with Quotas to Production Units :

Assigning quotas to production units does simplify the task of the planner because in successive iterations the number of activities involved in the production of i th product will be unchanged. Under fixed quota assignments the following proposition will hold.

**PROPOSITION 1:** When the plan is improving, ovarall savings in the value of the output must exceed the difference between the feedback (differential) costs of any two successive periods.

The verification of this statement is easy. From definition 5, we have :

$$\sum_{i,l} c_i^{It+1} q_i^{It+1} \leqslant \sum_{i,l} c_i^{It} q_i^{It} (t = 0, 1, 2, ..., T)$$

For every conditionally optimal plan of all periods, the following identity holds :

$$\sum_{i,l} c_i^{lt} q_i^{lt} = \sum_{i,l} p_i^t q_i^{lt} - \sum_j r_j Q_j \quad (t = 0, 2, ..., T)$$

It follows that

$$\sum_{i,I} p_i^{t+1} q_i^{It+1} - \sum_j r_j^{t+1} Q_j \leq \sum_{i,I} p_i^t q_i^{It} - \sum_j r_j^t Q_j$$

Rearranging, and remembering the quota assignments will be unchange d for all periods, we have :

$$\Sigma (p_i^t - p_i^{t+1}) q_i \geqslant \Sigma (r_j^t - r_j^{t+1}) Q_j$$

and this is what proposition 1 claims.

The decision-rule for the production units depends upon the unit cost of production. Given the set of prices and scarcity valuations, units are expected to submit production proposals to the CPB. The simple decision rule would suggest that:

If 
$$\sum_{J} a_{ij}^{tt+2} p_{j}^{t+1} < \sum_{J} a_{ij}^{tt+1} p_{j}^{t}$$
 (i = 1,2...,n;L = 1,...,s),  
(t = 0,1,2,...,T)

the production unit will propose the new program.

If  $\sum_{J} a_{ij}^{It+2} P_j^{t+1} > \sum_{J} a_{ij}^{It+1} p_j^t$ 

The units will propose their old program.

## b) Planning Without Quotas to Production Units :

The proposition 1 will not hold for the case of no quota assignment because in the latter case there is no longer a unique relationship between the labour costs and scarcity valuations. Since each unit is told to produce as much as it can without violating the macro production target, and since productive

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capacities are different, each will offer programs with varying production levels. Hence,  $r_i$  will be dependent upon the volume of production by the units. Without utilizing scarcity valuations an algorithm is still possible, but it will be considerably more difficult than our first case (with quota assignment).

The general rule, again, is : (for all i,j,1)

If	$\sum\limits_{J} a_{ij}^{t+2} p_{j}^{t+1} < \sum\limits_{J} a_{j}^{It+1} p_{j}^{t}$	new proposal wil be made.
If	$\sum_{J} a_{ij}^{It+2} p_{j}^{t+1} > \sum_{J} a_{ij}^{It+1} p_{j}^{t}$	units will propose their old programs.

Further specification of decision-rule is possible :

(i) If production unit produces all output,  $q_i$ , with the new activity; the new activity will replace all other activities if;

$\Sigma a_{ii}^{1t+2} P_i^{t+1} \leqslant \Sigma a_{ii}^{1^* t+1} P_i^t$	where l* is the lo-
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	pproducing i th
	product.

(ii) If the production unit produces only a fraction of the total output,  $q_i$ , than in conformity with the general rule, new activity will replace the higher cost activities.

#### 7. A Description of Planning Procedure :

Let us now describe the planning routine for the first case where units are given specific quotas to produce. The second case will be dealt with in a later section.

We have established that scarcity valuations are related positively to labour costs. That is, the higher the scarcity valuations of resource the higher will be labour costs and vice versa. Proposition 1 indicates that proposals by the units will be judged on the basis of scarcity valuations. If two distinct proposals exist using the same scarce resource, the proposal with a lower "r" will be awarded the contract and that with the higher "r" will be rejected.

We have assumed that (by Assumption 2) at the initial stage there exists a plan which can produce the predetermined output with the given means. This initial plan will have a set of prices and scarcity valuations associated with it. The CPB, then, will iniate a procedure on the basis of this information. We should point out, however that if very ambitious output targets could be revised and altered, assumption 2 can be dispensed with. In this case, an initial feasible plan can be reached by an exchange of information between units and the CPB, the production units making an inventory of their productive capacities and informing the center accordingly. The CPB would then try to find a feasible plan, if this cannot be done, output targets will again be revised and the initial plan will be pursued. Since we have an initial feasible plan by assumption, the planning routine will be initiated in the following manner:

**Step 1**: The CPB has, at its disposal, a set of prices,  $p_i$ , and scarcity valuations,  $r_j$ , associated with the feasible programme. They will announce the prices and ask the units to prepare their production proposals based on these prices and the goal of minimum labour costs.

**Step 2:** The production units, given the prices, will be searching for new oppurtunities and possibilities to improve their initial programmes. Specifically, they will try to minimize the labour costs of producing their particular output. Obeying the decision-rules we have established earlier, they will either propose new programmes or ask for reaffirmation of their old programmes.

**Step 3**: The CPB, receiving these proposals, will have to check the overall plan for feasibility. This is because their announcing prices and asking units to minimize their outlays is the equivalent of relieving units from their earlier engagements and contracts. Practically this will amount to bidding the resources and the means of production away from the current users. This will immediately create a problem of feasibility. It would be extremely lucky if all proposals together were to constitute a feasible macroplan under such circumstances. In fact, a search for a feasible plan at this stage would be central to the planning procedure. We shall later discuss this search.

Step 4: The CPB will aggregate the proposals of the units and determine which means of production and which resources are in excess demand.

Step 5: If certain resources are in excess demand with respect to initial stocks available, this means that the new macroplan is not feasible. Some proposals will have to be rejected and new ones soliciated. When the feasible plan is found the cycle is complete. The whole procedure will be repeated to the desired degree of approximation with new set of prices from the new plan.

Now, before going into the evaluation of the procedure decribed.

**PROPOSITION 2:** If proposals by units, based upon the price information made available by the CPB, constitute of feasible plan, it is also a better plan.

This statement is a consequence of labour cost minimization. If, at new prices, an activity can produce the given quantity with lower labour costs, it will be proposed as their new programme; otherwise the old program will be asked for approval. If at least one unit has a new proposal, then; by definition 5, we have an improving plan that the overall programme also is feasible.

The importance of proposition 2 is that it indicates the direction in which our algorithm should move. In particular, it suggests that we should shift our attention from the optimal plan to feasible plans at this stage of the planning routine. To find feasible plans becomes an important feature of this procedural model.

#### 8. Evaluation of Proposals at the CPB:

The above explanations make it clear the most difficult task in completing the plan falls to the CPB. From the proposed programs they must build a consistent and feasible macro-plan. How is this to be achieved?

The problem now is to find a feasible programme (or programmes) from the set of possible feasible programmes. The most obvious nevertheless clumsy answer, would be to check every

combination for feasibility and compare their labour costs. One realizes that this would be a very tedious and cumbersome process and without the use of high-powered computers the task would be insurmountable.

Another way out may be suggested. At step 3, the CPB may check the proposed programmes for feasibility. If a plan is feasible, it is also a better plan and the CPB would then have an easy solution. They would approve all proposals (old and new) for the current plan period. As indicated earlier, however, this event would have been a rare bit of extremely good luck. In general certain scarce resources will be in excess demand. The CPB, then, should follow certain steps in order to locate the feasible plan.

Step a: If initial proposals do not constitute a feasible plan the CPB will fallow step 4, namely, it has to aggregate the unit proposals and see which restrictions are violated.

**Step b**: At this stage, certain proposal would have to be rejected by the CPB in order to achive feasibility. The CPB ascends proposals which make use of scarce resources and ranks them with respect to their scarcity valuations.

**Step c**: The CPB will reject the proposals with the highest scarcity valuations until the restriction on the j th scarce resource is not exceeded.

Step d: When a new proposal is rejected, the old program of the same unit will be instituted. Since both new and old programmes produce the same quantity, step d will assure that when scarcities are obeyed, plan will not fall short of producing the target output.<sup>(5)</sup>

The logic of the steps just proposed is explained on the following table :

For convenience, only two restrictions are violated,  $Q_j$  and  $Q_k$ . Assume that four activities use  $Q_j$  and three activities use  $Q_k$ . We calculate the scarcity valuation  $(r_{ij})$  for each activity and form the matrix of scarcity valuations as shown in the table 1.

<sup>(5)</sup> Each production unit will have at least one proposal at CPB's disposal. It is exactly one if in priod "t + 1" they propose their programmes of "t". Unit will have two proposals if they proposed a new programme in "t + 1" for approval. Their t-period programme will be kept in hand in case of rejection.

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Activity Scarce	Qj	Qk
Act. 1	r <sub>1j</sub>	r <sub>1k</sub>
Act. 2	r <sub>2j</sub>	r <sub>2k</sub>
Act. 3	r <sub>3j</sub>	r <sub>3k</sub>
Act. 4	r <sub>4j</sub>	r <sub>4k</sub>

Since only three activities using  $Q_k$ ,  $r_{4k}$  is zero. In general  $r_{ij}$  refers to the scarcity valuation of the j th resource in the ith activity.

In order to achieve feasibility in  $Q_j$  and  $Q_k$ , activities from each column will be cut and immediately be replaced by their old programmes at "t". The rejection of an activity will be based on the magnitude of  $r_{ij}$ . Let us see how the process will work from a numerical example. Assume that scarcity valuation yielded the following values. For feasibility, Activity 1 of  $Q_j$  and Activity 2 of  $Q_k$  will be rejected and be replaced by their old programmes. If  $Q_k$  obeys the restriction but  $Q_j$  is still excess demand, Activity 4 of  $Q_j$  will also have to be rejected and replaced by its old programme (activities which are rejected are shown in circles).

Scarce Resource Activity	Qj	Qk
Act. 1	0.62	0.09
Act. 2	0.23	0.81
Act. 3	0.18	0.32
Act. 4	0.45	0

The steps we have discussed so far do not yet assure the CPB of locating the feasible programme. It is still quite possible that all proposed activities may be rejected due to the fact that

may still violate the restrictions on  $Q_j$  and  $Q_k$ . When all proposals (new) are rejected for making use of  $Q_j$  and  $Q_k$ , we reach a feasible plan but this is the plan in period "t". If such is the case, all units will stick to their old programmes of "t" in "t+1".

**Step e**: Only if steps described above fail to find a feasible plan does it become necessary to check all possible programmes for feasibility. However, this can be done by taking advantage of the fact that the proposals with highest scarcity valuations contribute most to the labour costs and must be excluded first.

Step f: When the feasible plan is found, planners do not have to look for other feasible programmes and the search stops. Because, by Proposition 2, if a new feasible plan is found it is a better plan. At this point, the CPB will approve the unit programmes which comprise the feasible macro-plan and reject those which are not part of it. The feasible plan will be the "conditionally optimal" plan of "t+1".

#### 9. Properties of the Procedure :

The crucial part of the planning procedure, as we have seen, was completed at the CPB. The search for a feasible plan by the CPB occupied a prominent place in the entire routine. For this reason, we may call our plan "center - dominated". It closely resembles the method of material balances because of the crucial importance of "feasibility" in this procedure. The following results for the procedure can be given :

(i) From the proposals based upon price information of period-t, a plan for t + 1 is drawn in such a way that the value of the plan in "t + 1" cannot exceed the value of the plan in t. This was established by proposition 2. Hence, the plan is said to be improving.

(ii) The CPB's search for a feasible plan improves on the simplex method of linear programming by involving scarcity valuations as a criterion for search. It assures planners that the first feasible programme found with the procedure is a conditionally optimal programme. All other feasible plans are inferior, because proposals with higher unit labour costs are first excluded. (iii) The planning routine will come to an end when no new proposals are made or all new proposals are rejected by the CPB.

(iv) When the optimal plan is reached, we have :  $p_i^t = p_i^{t+1} = p_i^*$ 

where  $p^{\star_i}$  is the optimal price. Equivalently,  $c_i^{\ 1t}=c_i^{\ 1t+1}=c_i^{\ 1t+2}...=c_i^{\star 1}$ 

(v) Since macro-output targets for the economy are fixed by assumption 3 and there are quota assignments for production units, result 4 also implies that

$$\sum_{i,1} c_i^{I\iota} q_i^{I\iota} = \sum_{i,1} c_i^{I} q_i^{I}$$

Both  $p_i$  and  $c_i$  are bounded from below sets, a non-increasing sequence. Then we must have the value of the plan is also bounded from below and the minimum of socially necessary outlays exist. Lower bound for  $p_i$  and  $c_i$  is, of course, equal to the direct labour input which is some fraction of  $c_i$ . If so desired, there exists a stage t, at which the iterative procedure may be stopped satisfying the desired degree of approximation. If  $\varepsilon$  is an arbitrary positive number within which the difference of values of two successive plans are supposed to lie,

$$(S^t - S^{t+1}) < \varepsilon$$

Then, there exist all but finite number of iterations, t, which satisfy this condition.

(vi) The planning procedure is well-defined because the rules of feasibility are obeyed and at every step it is possible to determine the value of the plan, unit costs, prices and scarcity valuations.

(vi) There are a finite number of steps involved in finding the feasible plan because there are only a finite number of proposals, but the number of iterations which lead to an ovarall optimum is infinite.

## 1. Extensions of the Model:

A number of suggestions may be given at this point. We shall briefly consider only a few of the more important ones.

(i) The number of proposals made by every unit may be enlarged by including within the procedure the programs of past several years instead of only one period. In this case, our planning procedure is still, in principle, capable of handling the problem. However, the number of possible feasible programmes to be checked increases considerably.

(ii) The quota requirements four units may be dropped. But then another dimension has to be taken into consideration since dropping the programme of last year and adding the new proposal does not solve the problem of meeting the macro output targets. The CPB, now, has to check not only the violations of input requirements but also of output targets. This can be acheived by specifying the number of possible ways to produce the target output with the activities proposed. No doubt the number of possibilities will be further increased.

The crucial problem with the algorithm, without quota assignments to the units, is that scarcity valuations can no longer quide the selection of activities in a systematic their feasibility and their values must be compared. This case corresponds more or less to the simplex method of linear programming.

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## ÖZET

# MERKEZDEN PLANLI VE KITLIK İÇEREN BİR EKONOMİDE PLAN SÜRECİ

Bu çalışmada emek-değer temelli fiatlara dayalı bir plân modelinin özellikleri; model dinamik bir plân yöntemi olrak düşünüldüğünde dinamik özellikler; her aşamada gelişen (daha iyilenen) bir programın nitelikleri araştırılmağa çalışılmıştır.

İlk bölümde, gerek batı kökenli gerekse sosyalist ülke iktisatçılarının türetmiş olduğu modellere değinilmiş ve bunların ortak özellikleri araştırılmıştır. Farklı kaynaklı olan bu modelleri a) Hiyerarşik modeler, b) Tatonnement süreçler olarak ayırmak mümkündür. Bu çalışmadaki model, hiyerarşik modeller çerçevesindedir.

Üzerinde iterative bir plân sürecinin kurulmuş olduğu model Sovyet İktisatçısı Novozhilov'undur. Model dinamik bir hale dönüştürülüp bir plân yöntemi olarak düşünüldüğünde, her adımda optimuma yaklaşan bir süreç tanımlanabilmektedir. Bu süreç, iyi tanımlanmıştır, emek-değer temellidir ve plân yöntemlerinin istenir temel özelliklerine sahiptir.